

# Form Factor Relations for Heavy-to-Light Transitions\*

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## Abstract:

Assuming simple properties of the spectator particle in weak decays the form factors of hadronic current matrix elements are shown to be related to a single universal function. The Isgur-Wise result for heavy-to-heavy transitions follows as well as similar relations for heavy-to-light decay processes. The approximation should hold for total energies of the final particle large compared to the confinement scale. A comparison with experimentally determined  $D$ -decay form factors and QCD sum rule results for  $B$ -decays is very encouraging.

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The dynamical content of hadronic current matrix elements is described by Lorentz-invariant form factors. Knowledge of these form factors is essential for the description of semileptonic and nonleptonic weak decay processes and in particular for the experimental determination of the fundamental Kobayashi-Maskawa matrix elements. For transitions containing one (infinitely) heavy quark in the initial and another heavy quark in the final state

(heavy-to-heavy transitions) the number of relevant form factors is greatly reduced. For instance, in the limit  $m_c \gg \bar{\Lambda}, m_b \gg \bar{\Lambda}$  (with  $\bar{\Lambda} \simeq m_D - m_c, m_B - m_b$ ) the 6 form factors describing  $\bar{B} \rightarrow D$  and  $\bar{B} \rightarrow D^*$  vector and axial vector matrix elements are

are all related to a single unknown form factor, the Isgur-Wise function [1]. Even though  $m_c$  is not really large and leads to sizeable  $1/m_c$  corrections the Isgur-Wise relations provide for a good starting point for more detailed investigations based on the heavy quark effective theory (HQET) [2].

For heavy to light transitions such as  $B \rightarrow \pi, B \rightarrow \rho$ , on the other hand, similar relations among the form factors cannot be derived by using the heavy quark limit. The heavy quark symmetries are not applicable and the number of independent form factors is not reduced.

Nevertheless, in the present note I will show that interesting relations among heavy-to-light form factors can be obtained, if use is made of a constituent quark picture.

Let us consider the four momentum of a  $B$ -meson of mass  $m_B$  and velocity  $v^B$  and divide it into the momenta of the constituent  $b$ -quark  $p_b^B$  and the spectator  $p_{sp}^B$

$$\begin{aligned} P^B &= p_b^B + p_{sp}^B \\ p_b^B &= \epsilon_b^B v^B + k^B, \quad p_{sp}^B = \epsilon_{sp}^B v^B - k^B \\ \epsilon_b^B + \epsilon_{sp}^B &= m_B \end{aligned} \tag{1}$$

Here,  $\epsilon_b^B$  and  $\epsilon_{sp}^B$  denote the constituent masses of  $b$ -quark and spectator within the  $B$ -meson, respectively. The bound state dynamics is contained in the distribution

function for the off-shell momentum  $k^B$ , i.e. the wave function of the  $B$ -meson. The four momentum of the final particle  $F$

(with mass  $m_F$  and velocity  $v^F$ ) emitted in the weak process is similarly decomposed. It contains the spectator particle plus an  $u$ -quark in a  $b \rightarrow u$  transition or a  $c$ -quark in a  $b \rightarrow c$  decay:

$$\begin{aligned} P^F &= p_{u,c}^F + p_{sp}^F \\ p_{u,c}^F &= \epsilon_{u,c}^F v^F + k^F, \quad p_{sp}^F = \epsilon_{sp}^F v^F - k^F \\ \epsilon_{u,c}^F + \epsilon_{sp}^F &= m_F \end{aligned} \tag{2}$$

At this point I will make two dynamical assumptions:

i) In the rest system of a hadron the distribution of the components of  $k^\mu$  are strongly peaked with a width corresponding to the confinement scale. (The values of the spectator masses  $\epsilon_{sp}^{B,F} \ll \epsilon_b^B$  are chosen such that the peak is at  $k_\mu^{B,F} = 0$ ). This assumption is plausible considering the numerical value of the average  $b$ -quark root mean square longitudinal momentum  $\sqrt{\langle p_z^2 \rangle} \approx 0.4$  GeV as obtained from QCD sum rules [3]. It implies the dominance of soft gluon effects over hard gluon emission before and

after the weak process.

ii) During the weak transition the spectator — whatever it consists of — retains its momentum and spin. This requirement is clearly satisfied in any Fock space calculation of the transition amplitude.

As a consequence of ii) the momenta  $k^F$  and  $k^B$  are correlated in the weak process:

$$k^B - \epsilon_{sp}^B v^B = k^F - \epsilon_{sp}^F v^F \tag{3}$$

While initial and final wave functions have their peaks at

$\bar{k}^B = \bar{k}^F = 0$ , due to eq. (3) the integrand of the transition amplitude has a maximum (with a width  $\lesssim \bar{\Lambda}$ ) for values of  $\bar{k}^B$  and

$\bar{k}^F$  different from zero.

Still, these values are such that — in the rest system of the  $B$ -meson — both sides of eq. (3) stay of order  $\epsilon_{sp}^{B,F} \ll \epsilon_b^B$ , even for the most energetic transitions. To illustrate this, we consider, as an example, a Gauss form of initial and final wave functions.  $\bar{k}_B$  and  $\bar{k}_F$  are then determined by the minimum of

$$\alpha \left[ 2(k^F \cdot v^F)^2 - (k^F)^2 \right] + 2(k^B \cdot v^B)^2 - (k^B)^2 \tag{4}$$

where  $\alpha \approx 1$  denotes the ratio of the square of final

and initial particle radii. In the  $B$ -meson rest system and for the spatial momentum of the final particle pointing in  $z$ -direction, the result for  $\bar{k}^B$  is<sup>1</sup>

$$\begin{aligned} (\bar{k}^B)_0 &= \frac{1}{2}\epsilon_{sp}^B - \frac{1}{2}\frac{m_F}{E_F}\epsilon_{sp}^F, & (\bar{k}^B)_z &= -\frac{1}{2}\frac{P_F}{E_F}\epsilon_{sp}^B \\ (\bar{k}^B)_\perp &= 0. \end{aligned} \quad (5)$$

$E_F$  and  $P_F$  denote energy and momentum of the final particle. Thus, the relevant  $b$ -quark space momenta active in the transition and the  $b$ -quark energy variations are small even in a transition with large energy release! We can neglect  $\bar{k}^B$  compared to the  $b$ -quark mass.

Using eq. (2,3), one can now estimate the momentum range of the generated  $u$ - or  $c$ -quark and finds that — in the  $B$ -meson rest frame — it is peaked around

$$\begin{aligned} (\bar{p}_{u,c})_0 &= E_F(1 + O(\epsilon_{sp}^{F,B}/E_F)) \\ (\bar{p}_{u,c})_z &= P_F(1 + O(\epsilon_{sp}^{F,B}/E_F)) \end{aligned} \quad (6)$$

Thus, the  $u$ - or  $c$ -quark momenta effectively determining the weak transition amplitude lie close to the 4-momentum of the

final particle — apart from corrections of order  $\epsilon_{sp}/E_F$ .

From this result it is easy to obtain form factor relations in the limit where  $(\epsilon_{sp}^{F,B})^2$  and the average

transverse quark momentum squared are small compared to  $E_F^2$ : The transition matrix element of the weak current is simply

proportional to the  $c$ -number matrix element  $T^\mu$

$$T^\mu = \left( \bar{u}_{u,c}^{s'}(\vec{P}_F, m_{u,c}) \gamma^\mu (1 - \gamma_5) u_b^s(\vec{0}, m_b) \right) L_{s',s}. \quad (7)$$

Here  $m_{u,c}$  and  $m_b$  are current masses of the  $u$  or  $c$ -quark and the  $b$ -quark, respectively. The energy  $\bar{p}_{u,c}^0 \simeq \sqrt{\vec{P}_F^2 + \vec{k}_\perp^2 + m_{u,c}^2}$  must be identified with  $E_F$ . The  $L_{s',s}$  are the elements of a  $2 \times 2$  spin matrix with  $L = \mathbb{1}$  for

$B$ -decays to a pseudoscalar state, e.g. the  $\pi$ -meson, and  $L = \sigma \cdot \vec{e}$  for  $B$ -decays to a vector particle polarized

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<sup>1</sup>For simplicity, we put  $\alpha = 1$ . A change of this value or a different choice ( $> 1$ ) for the factors 2 in the expression (4) do not invalidate the conclusions given below.

in  $\vec{e}$  direction. The form (7) satisfies the requirement that the spin components of the spectator particle remain unaffected in the decay process.

A comparison of (7) with the conventional form factor decomposition [4] of the current matrix element gives for a transition to a pseudoscalar particle (the  $\pi$  or the  $D$ , for instance)

$$\begin{aligned} F_1(q^2, m_F) &= \left(1 + \frac{m_{u,c}}{m_B}\right) R_{u,c}^B(q^2, m_F) \\ F_0(q^2, m_F) &= \left(1 + \frac{m_{u,c}}{m_B} - \frac{q^2}{m_B^2 - m_F^2} \left(1 - \frac{m_{u,c}}{m_B}\right)\right) R_{u,c}^B(q^2, m_F). \end{aligned} \quad (8)$$

$R_{u,c}^B(q^2, m_F)$  is an unknown universal function depending not only on  $q^2$  but also on  $m_F$  and the flavor of the outgoing quarks (and on  $m_B$ ).

For transitions to a vector particle (the  $\rho$  or  $D^*$  for instance) I find with  $E_F = \frac{1}{2m_B}(m_B^2 + m_F^2 - q^2)$

$$\begin{aligned} V(q^2, m_F) &= \frac{m_B + m_F}{m_B} R_{u,c}^B(q^2, m_F) \\ A_1(q^2, m_F) &= 2 \frac{m_{u,c} + E_F}{m_B + m_F} R_{u,c}^B(q^2, m_F) \\ A_2(q^2, m_F) &= \frac{m_B + m_F}{m_B} \frac{E_F - \frac{m_F^2}{m_B} + m_{u,c} \left(1 + \frac{m_F}{m_B}\right)}{m_F + E_F} R_{u,c}^B(q^2, m_F) \\ A_0(q^2, m_F) &= \frac{m_{u,c} + m_B}{m_B} R_{u,c}^B(q^2, m_F). \end{aligned} \quad (9)$$

In the limit of large  $m_c$  one can use the approximation  $m_D = m_{D^*} = m_c$  i.e. apply the spin symmetry of HQET valid for heavy-to-heavy transitions. Eqs. (8) and (9) then give

$$\begin{aligned} F_1^{HH}(q^2) &= \left(1 + \frac{m_D}{m_B}\right) R_c^B(q^2, m_F) \\ F_0^{HH}(q^2) &= A_1^{HH}(q^2) = \left(1 - q^2/(m_B + m_D)^2\right) F_1^{HH}(q^2) \\ V^{HH}(q^2) &= A_2^{HH}(q^2) = A_0^{HH}(q^2) = F_1^{HH}(q^2). \end{aligned} \quad (10)$$

Thus the well-known heavy-to-heavy form factor relations [5] based on the heavy quark limit [1] are contained in (8) and (9).

For  $b \rightarrow u$  transitions we can set  $m_u = 0$  and find

$$F_1^{HL}(q^2, m_F) = R_u^B(q^2, m_F)$$

$$\begin{aligned}
F_0^{HL}(q^2, m_F) &= \left(1 - \frac{q^2}{m_B^2 - m_F^2}\right) F_1^{HL}(q^2, m_F) \\
V^{HL}(q^2, m_F) &= \left(1 + \frac{m_F}{m_B}\right) F_1^{HL}(q^2, m_F) \\
A_1^{HL}(q^2, m_F) &= \frac{1 + \frac{m_F^2}{m_B^2}}{1 + \frac{m_F}{m_B}} \left(1 - \frac{q^2}{m_B^2 + m_F^2}\right) F_1^{HL}(q^2, m_F) \\
A_2^{HL}(q^2, m_F) &= \left(1 + \frac{m_F}{m_B}\right) \left(1 - \frac{2m_F/(m_B + m_F)}{1 - q^2/(m_B + m_F)^2}\right) F_1^{HL}(q^2, m_F) \\
A_0^{HL}(q^2, m_F) &= F_1^{HL}(q^2, m_F).
\end{aligned} \tag{11}$$

Remarkably, the heavy-to-light form factor relations are not very different from to the heavy-to-heavy form factor relations (10). But, of course, here the dependence of  $F_1$  on  $m_F$  has

to be taken into account due to the lack of spin symmetry in the final state.

Of particular interest is the fact that the longitudinal form factor  $F_0$  and the transverse form factor  $A_1$  again behave differently from the remaining form factors. This result is strongly supported by a recent detailed QCD sum rule calculation of  $B \rightarrow \rho$  form factors by P. Ball [8]. She found a strong difference between the  $q^2$ -dependence of  $A_1$  and the other  $B \rightarrow \rho$  form factors. Similar results have been obtained in ref. [6], [7]. Ref. [7] indicates that also the form factor  $A_0$  satisfies eq. (11).

The differential branching ratio for a semileptonic decay to a vector particle using Eq. (9) is now (in a more general notation and neglecting the lepton mass):

$$\begin{aligned}
\frac{d BR(q^2)}{dq^2} &= \frac{G_F^2}{192\pi^3} \frac{\lambda(q^2)}{m_I^5} \tau_I |V_{fi}|^2 \left(R_f^I(q^2, m_F)\right)^2 \cdot \left(S_T(q^2) + S_L(q^2)\right) \\
\lambda(q^2) &= [(m_I + m_F)^2 - q^2]^{1/2} \cdot [(m_I - m_F)^2 - q^2]^{1/2}.
\end{aligned} \tag{12}$$

Here  $V_{fi}$  denotes the relevant Cabibbo-Kobayashi-Maskawa matrix element,  $G_F$  the Fermi constant and  $\tau_I$  the lifetime of the initial pseudoscalar meson.  $S_T(q^2)$  and  $S_L(q^2)$  are the transverse and longitudinal polarization contributions, respectively:

$$\begin{aligned}
S_T(q^2) &= 2q^2 \left[ \lambda^2(q^2) + 4m_I^2(m_f + E_F(q^2))^2 \right] \\
S_L(q^2) &= \left[ (m_I^2 - m_F^2)(m_I + m_f) - q^2(m_I - m_f) \right]^2.
\end{aligned} \tag{13}$$

$m_f$  denotes the current mass of the emitted quark active in the process. The differential branching ratio for a

decay to a pseudoscalar particle is obtained from (12) by replacing the sum  $S_T + S_L$  by

$$S_P(q^2) = \lambda^2(q^2)m_I^2 \left(1 + \frac{m_f}{m_I}\right)^2. \quad (14)$$

Eqs. (8-14) are expected to hold to a good approximation for  $E_F \gg \epsilon_{sp} \approx 0.35$  GeV. The only unknown is the function  $R_f^I(q^2, m_F)$ .

The assumptions leading to the result (8-14) are rather general. It should hold or approximately hold in all

quark model calculations which treat the spectator the same way as done here and use the relativistic Dirac-spinor structure. An interesting publication by the Orsay group [9] deals with an explicit semi-relativistic wave function model which gives a

decreasing  $q^2$ -behaviour of  $A_1(q^2)/V(q^2)$  and reproduces for large  $m_F$  the Isgur-Wise relations. Their formulae differ, however, for heavy-to-light transitions from the ones found here since in their model the light quark is not treated in a fully relativistic manner. Another model worth mentioning here is the one by Faustov and Galkin [10].

One may be hesitant to apply the result (8,9) for  $D$ -decays (replacing  $m_B$  by  $m_D$  and  $m_u$  by  $m_s$ ) because of the relatively low energies of the final particles involved. Let us nevertheless try it. Using  $m_s = m_{D_s} - m_{D^+} = 0.10$  GeV one obtains for  $D \rightarrow K^*$  transitions at  $q^2 = 0$  the form factor values shown in Table I.

**Table I:**  $D \rightarrow K^*$  form factors at  $q^2 = 0$

	theory (Eq. (9))	experiment [12]
$V(0)$	$1.00 \cdot \delta$	$1.16 \pm 0.16$
$A_1(0)$	$0.61 \cdot \delta$	$0.61 \pm 0.05$
$A_2(0)$	$0.42 \cdot \delta$	$0.45 \pm 0.09$

For the ratio of form factors the agreement with experiment<sup>2</sup> is surprisingly good.

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<sup>2</sup>The data at  $q^2 = 0$

are extracted from integrated rates assuming single pole formulae. Thus, they are not completely free of theoretical uncertainties.

Moreover, I will show below that  $\delta$  defined by  $\delta = R_s^D(0, m_{K^*})/0.67$  can be estimated and turns out to be very close to one.

For  $B$ -decays to light particles there are not yet

experimental data available to test Eq. (11). One can, however, compare form factor ratios from (11) with the explicit QCD sum rule calculations of ref. [8] and ref. [11]. Table II shows as

representative examples the  $B \rightarrow \rho$  transition form factors at  $q^2 = 0$  and at  $q^2 = 8 \text{ GeV}^2$ . Noticeably, there is agreement with the QCD sum rule result. In particular, in all three calculations the ratio  $A_1/V$  falls off with  $q^2$ .

**Table II:**  $B \rightarrow \rho$  form factors at  $q^2 = 0$  and  $q^2 = 8 \text{ GeV}^2$

	theory (Eq. (11))	ref. [8]	ref. [11]
$A_1/V _{q^2=0}$	0.78	$0.83 \pm 0.32$	$0.86 \pm 0.23$
$A_2/V _{q^2=0}$	0.75	$0.67 \pm 0.40$	
$A_1/V _{q^2=8 \text{ GeV}^2}$	0.56	$0.50 \pm 0.19$	$0.60 \pm 0.19$
$A_2/V _{q^2=8 \text{ GeV}^2}$	0.67	$0.66 \pm 0.39$	

Encouraged by the above success one can go further and can try to relate heavy-to-heavy with heavy-to-light form factors. This requires, however, a new dynamical assumption referring to the formation of the final particle, e. g. a fixing of

initial and final wave functions. As a first attempt let us simply ignore the different structure and radii of the final hadrons of a given spin (say a  $\rho$  and a  $D^*$ ). Under this condition  $R_{u,c}^B(q^2, m_F)$  depends only on the velocity of the outgoing hadron apart from the explicit dependence on the quarks contained in the Dirac spinors of Eq. (7). Since  $R$  does scale with  $m_B^{1/2}$ , Eq. (7) together with

dimensional arguments allow to write  $R$  in the form

$$R_{u,c}^B(q^2, m_F) = \frac{1}{2} \left( \frac{m_B}{E_F + m_{u,c}} \right)^{1/2} (1+y)^{1/2} \xi(y)$$

$$y = \frac{E_F}{m_F} = v_F \cdot v_B = \frac{m_B^2 + m_F^2 - q^2}{2m_B m_F}. \quad (15)$$

Here  $\xi(y)$  is now a universal function solely dependent on  $y$ , i.e. the same for  $\bar{B} \rightarrow D^*$ ,  $\bar{B} \rightarrow \rho$  and  $D \rightarrow K^*$  transitions. The prefactors in (15) have been chosen



in such a way that  $\xi(y)$  is just the Isgur-Wise function as can be seen by comparing (15) with (10) and using the conventional definition of this function for heavy-to-heavy transitions. Clearly, a direct practical use of comparing a  $b \rightarrow c$  with a  $b \rightarrow u$  or  $c \rightarrow s$  transition can only be made if the values of  $y$  considered belong to the physical region of both processes. Moreover, good results can only be expected if  $E_F \gg \epsilon_{sp}^{B,F}$  holds for the heavy-to-light transition.

As a simple test for the applicability of (15) one can take the numerical value of the Isgur-Wise function for the  $\bar{B} \rightarrow D^*$  transition at a given value of  $y$

in order to obtain the form factors for  $\bar{B} \rightarrow \rho$  decays at the corresponding  $q^2$  value. For  $y = 1.5$  (i.e.  $q^2 = 0$  for the  $\bar{B} \rightarrow D^*$  decay) the corresponding momentum transfer in the  $\bar{B} \rightarrow \rho$  transition is  $q^2 = 16.3 \text{ GeV}^2$ . Taking  $V_{cb} \cdot \xi(1.5) = 0.023 \pm 0.002$  [13] and  $V_{cb} = 0.040 \pm 0.003$  [14], Eq. (15) gives  $R_u^B(16.3, m_\rho) = 0.97 \pm 0.11$ . From (11) one then gets the  $B \rightarrow \rho$  form factor values shown in

Table III. Remarkably, the theoretical numbers are in agreement with the values obtained from the plots in ref. [8] and ref.

[11].

**Table III:**  $B \rightarrow \rho$  form factors at  $q^2 = 16.3 \text{ GeV}^2$

	theory (Eq. (11))	ref. [8]	ref. [11]
$V^{B \rightarrow \rho}(16.3)$	$1.11 \pm 0.13$	$0.96 \pm 0.32$	$1.55 \pm 0.50$
$A_1^{B \rightarrow \rho}(16.3)$	$0.37 \pm 0.04$	$0.30 \pm 0.06$	$0.50 \pm 0.05$
$A_2^{B \rightarrow \rho}(16.3)$	$0.60 \pm 0.07$	$0.52 \pm 0.26$	

By using the equivalent of (15) for  $D$ -decays

$$R_s^D(q^2, m_F) = \frac{1}{2} \left( \frac{m_D}{E_F + m_s} \right)^{1/2} (1 + y)^{1/2} \xi(y) \quad (16)$$

it is also possible to get from  $\xi(y)$  information on  $D \rightarrow K^*$  transitions. Choosing  $y = 1.28$  which corresponds to  $q^2 = 0$  for the  $D \rightarrow K^*$  transition and taking  $V_{cb} \cdot \xi(1.28) = 0.029 \pm 0.003$  [13], Eq. (16) leads to  $R_s^D(0, m_{K^*}) = 0.67 \pm 0.08$ . Thus, the quantity  $\delta$  defined earlier is obtained to be  $\simeq 1$  giving close agreement between theory and experiment in

$D$ -decays<sup>3</sup>.

Heavy-to-light current matrix elements are also needed in the calculation of non-leptonic and Penguin-induced matrix elements [4]. Of recent interest [9, 16] are the decays  $B \rightarrow K^{(*)}J/\psi$  which are given — in factorization approximation — by the  $B \rightarrow K^{(*)}$  form factors [4, 9, 16]. For the calculation of the polarization of  $K^*$  one needs only the ratio of form factors (at  $q^2 = m_{J/\psi}^2$ ). It can be directly obtained from (13). For the longitudinal polarization  $\rho_L$  one gets  $\rho_L = 0.41$  not in accord with the most recent value  $\rho_L = 0.66 \pm 0.1 \pm 0.1$  [17] or the even larger values of

previous Argus and Cleo results [18]. I do not consider the result for the longitudinal polarization as an

argument against (9). Factorization is an approximate concept [4], and the longitudinal polarization involving the interference of  $S$  and  $D$  waves is particular sensitive to final state interactions. The small class II transitions can always get corrections from the stronger class I transitions to (virtual) intermediate  $D^{(*)}\bar{D}_s^{(*)}$ -like states turning into  $K^*J/\psi$ .

For Penguin-induced processes like  $\bar{B} \rightarrow K^{*''}\gamma''$

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<sup>3</sup>A more direct way to obtain  $R_u^B$  and  $R_s^D$  from (15,16) is to extract  $R_c^B(q^2, m_{D^*})$  from the measured differential branching ratio [15] according to Eq. (12).

Isgur and Wise [19] derived relations between the corresponding form factors and the form factors of semi-leptonic decays. In particular, the magnetic moment form factor

$F^{magn.mom.}(q^2)$  — as defined for instance in ref. [11] — is related to the form factors  $A_1(q^2)$  and  $V(q^2)$ :

$$F^{magn.mom.}(q^2) = \frac{m_I + m_F}{2m_I} A_1(q^2) + \frac{V(q^2)}{2m_I} \frac{q^2 + m_I^2 - m_F^2}{m_I + m_F} \quad (17)$$

Originally derived for  $q^2 \approx q_{max}^2$  this equation is also supposed to hold for small  $q^2$  since — according to Burdman and Donoghue [20] — hard perturbative contributions may be neglected. QCD sum rule calculations [11] indeed show that eq. (17) holds to good accuracy for all momentum transfers. The present investigation supports this result. Moreover, for sufficiently large  $E_F$  one gets using (8,9)

$$F^{magn.mom.}(q^2, m_F) = F_1(q^2, m_F) = \left(1 + \frac{m_f}{m_I}\right) R_f^I(q^2, m_F) \quad (18)$$

thus providing for a simple connection between the branching ratios of radiative and semileptonic decays.

The formulae given in this paper give a handle on heavy-to-light matrix elements. Eqs. (8-14) combined with constraints from dispersion theory [21]

will be useful for the determination of the Kobayashi-Maskawa matrix element  $V_{ub}$ .

In addition — but with less rigor — one can make use of the universality property expressed in (15, 16): Taking a dispersion theoretic formula for the vector form factor  $R(q^2)$  in (12) and fitting the

corresponding parameters to a single decay mode many predictions can be made. However, more work is necessary to get a precise control of

the theoretical errors.

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